CCGPS
Frameworks
Student Edition

Mathematics

Fourth Grade Unit Four
Multiplying and Dividing Fractions

Dr. John D. Barge, State School Superintendent
"Making Education Work for All Georgians"

Unit 4 Organizer
Multiplying and Dividing Fractions

TABLE OF CONTENTS

OVERVIEW

In this unit students will:
- Understand a fraction *a/b* as a multiple of 1/*b*. (*for example: model the product of ¾ as 3 x ¼*).
- Understand representations of simple equivalent fractions
- Understand a multiple of *a/b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number.
- Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.
- Multiply a whole number by a fraction

To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the tasks listed under "Evidence of Learning" be reviewed early in the planning process. A variety of resources should be utilized to supplement. The tasks in these units illustrate the types of learning activities that should be utilized from a variety of sources.

STANDARDS FOR MATHEMATICAL CONTENT

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
MCC4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

    a. Understand a fraction *a/b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*

    b. Understand a multiple of *a/b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*

    c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ENDURING UNDERSTANDINGS

- Fractions can be represented visually and in written form.
- Fractional amounts can be added and/or multiplied.
- Multiplication by a fraction is similar to division of whole numbers.
- If given a whole set, we can determine fractional amounts
- If given a fractional amount we can determine the whole set

ESSENTIAL QUESTIONS

- How are multiplication, division, and fractions related?
- How can fraction represent parts of a set?
- How can I be sure fractional parts are equal in size?
- How can I find equivalent fractions?
- How can I model the multiplication of a whole number by a fraction?
- How can I multiply a set by a fraction?
- How can I multiply a whole number by a fraction?
- How can I represent a fraction of a set?
- How can I represent fractions in different ways?
- How can I represent multiplication of a whole number?
- How can I write a fraction to represent a part of a group?
- How can we model answers to fraction problems?
- How can we use fractions to help us solve problems?
- How can we write equations to represent our answers when solving word problems?
- How do we determine a fractional value when given the whole number?
- How do we determine the whole amount when given a fractional value of the whole?
- How does the number of equal pieces affect the fraction name?
- How is multiplication of fractions similar to division of whole numbers?
- How is multiplication of fractions similar to repeated addition of fraction?
- What do the numbers (terms) in a fraction represent?
- What does it mean to take a fraction portion of a whole number?
- What is the relationship between multiplication by a fraction and division?
- What is the relationship between the size of the denominator and the size of each fractional piece (i.e. the numerator)?
- What strategies can be used for finding products when multiplying a whole number by a fraction?
- Which problem solving strategies can we use to solve this problem?

CONCEPTS/SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Identify and give multiple representations for the fractional parts of a whole (area model) or of a set, using halves, thirds, fourths, sixths, eighths, tenths and twelfths.
- Recognize and represent that the denominator determines the number of equally sized pieces that make up a whole.
- Recognize and represent that the numerator determines how many pieces of the whole are being referred to in the fraction.
- Compare fractions with denominators of 2, 3, 4, 6, 10, or 12 using concrete and pictorial models.
- Understand repeated addition is one way to model multiplication, repeated subtraction is one way to model division
- Add and subtraction fraction with a common denominator
- Be able to decompose a whole into fractional parts. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

Teachers should present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for elementary children. One has activities to help students more fully understand and retain new vocabulary. (i.e. The definition for *dice* actually generates rolls of the dice and gives students an opportunity to add them.) **Note – At the elementary level, different sources use different definitions. Please preview any website for alignment to the terminology of the CCGPS.**

http://www.teachers.ash.org.au/jeather/maths/dictionary.html
http://intermath.coe.uga.edu/dictnary/

The terms below are for teacher reference only and are not to be memorized by the students.

- fraction
- denominator
- equivalent sets
- improper fraction
- increment
- mixed number
- numerator

- proper fraction
- term
- unit fraction
- whole number

STRATEGIES FOR TEACHING AND LEARNING

- Students should be actively engaged by developing their own understanding.
- Mathematics should be represented in as many ways as possible by using graphs, tables, pictures, symbols, and words.
- Interdisciplinary and cross curricular strategies should be used to reinforce and extend the learning activities.
- Appropriate manipulatives and technology should be used to enhance student learning.
- Students should be given opportunities to revise their work based on teacher feedback, peer feedback, and metacognition which includes self-assessment and reflection.
- Students should write about the mathematical ideas and concepts they are learning.
- Books such as *Fractions and Decimals Made Easy* (2005) by Rebecca Wingard-Nelson, illustrated by Tom LaBaff, are useful resources to have available for students to read during the instruction of these concepts.
- Consideration of all students should be made during the planning and instruction of this unit. Teachers need to consider the following:
  - What level of support do my struggling students need in order to be successful with this unit?
  - In what way can I deepen the understanding of those students who are competent in this unit?
  - What real life connections can I make that will help my students utilize the skills practiced in this unit?

EVIDENCE OF LEARNING
By the conclusion of this unit, students should be able to demonstrate the following competencies:
- Represent fractions of a set
- Represent fractional proportions of a whole numbers
- See a relationship between multiplying fractions and dividing whole numbers
- Solve story problems that involve multiplying a whole number by a fraction
- Apply rules of multiplication of whole numbers to multiplying fractions using a visual model
- Rewrite fractions as multiples of the unit fraction of the same denominator
- Multiply a fraction by a whole number using a visual model.
- Use visual fraction models to solve problems involving multiplication of a fraction by a whole number whole number
- Use equations to represent problems involving the multiplication of a fraction by a whole number

TASKS

The following tasks represent the level of depth, rigor, and complexity expected of all fourth grade students. These tasks or tasks of similar depth and rigor should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them. While some tasks are identified as a performance task, they also may be used for teaching and learning.

Scaffolding Task	Constructing Task	Practice Task	Performance Tasks
Tasks that build up to the constructing task.	Constructing understanding through deep/rich contextualized problem solving tasks	Games/activities	Summative assessment for the unit

Task Name	Task Type *Grouping Strategy*	Content Addressed
A Bowl of Beans	Scaffolding Task *Partner/Small Group Task*	Multiply a whole number by a fraction, represent fractional products
Birthday Cake!	Scaffolding Task *Individual/Partner Task*	Determine the fraction of a set, determine the set when given a fraction
Fraction Clues	Constructing Task *Individual/Partner Task*	Determine the fraction of a set, determine the set when given a fraction, find equivalent fractions
Area Models	Scaffolding Task *Individual/Partner Task*	Represent multiplication of fractions
Fraction Pie Game	Practice Task *Individual/Partner Task*	Write number sentences to show multiplication of fractions.
Birthday Cookout	Constructing Task *Individual/Partner Task*	Solve story problems that involve the multiplication of fractions
Fraction Farm	Performance Task *Individual/Partner Task*	Solve story problem that involves the multiplication of a whole number by a fraction
Culminating Task: Land Grant	Culminating Task *Individual/Partner Task*	Multiply whole numbers by fractions and represent products

SCAFFOLDING TASK: A Bowl of Beans

Adapted from Riddle Math: Using Student-Written Riddles to Build Mathematical Power (2001) by Carl M. Sherrill

STANDARDS FOR MATHEMATICAL CONTENT

MCC4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction *a/b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*

b. Understand a multiple of *a/b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Before assigning the task, students should have had several experiences with finding a fraction of a set. Also, students should have some degree of fluency with multiplication facts with a product of 24 or less.

Student solutions to the riddles should include a picture of the beans and an explanation using words and numbers. Some possible solutions are shown below:

Riddle #1

- I have 8 beans.
- $\frac{1}{4}$ of my beans are red.
- The rest are pinto beans.

Show my set of beans.
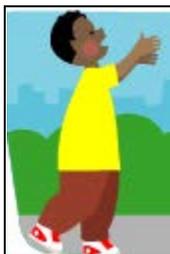
"I divided 8 beans onto 4 plates because I needed to find $\frac{1}{4}$ of the beans. There were 2 beans on each plate. I made the other six beans red beans. The answer to Riddle #1 is a set of 8 beans, 2 red beans and 6 pinto beans."

Riddle #2

- $\frac{1}{3}$ of my beans are red.
- I have 9 beans.
- $\frac{2}{3}$ of my beans are black.

Show my set of beans

"I divided 9 beans into 3 groups because I needed to find $\frac{1}{3}$ of the beans. There were 3 beans in each group. I know that 3 beans are $\frac{1}{3}$ of the total beans, so $\frac{2}{3}$ of the total beans must be 2 × 3 or 6 beans. So I made six beans black beans. The answer to Riddle #2 is a set of 9 beans, 3 red beans and 6 black beans."

"I divided 9 beans onto three plates because I needed to find $\frac{1}{3}$ of the beans. There were 3 beans on each plate. I know that $\frac{1}{3} + \frac{2}{3} = \frac{3}{3}$ so the rest of the beans must be $\frac{2}{3}$ of the total beans. So I made rest of the beans black beans. The answer to Riddle #2 is a set of 9 beans, 3 red beans and 6 black beans."

Riddle #3

- $\frac{1}{2}$ of my beans are black-eyed peas.
- $\frac{1}{4}$ of my beans are pinto beans.
- The rest are red.
- I have 12 beans.

Show my set of beans.

"I shared 12 beans in 2 groups because I needed to find $\frac{1}{2}$ of the beans. There were 6 beans in each group. I know that 6 beans are $\frac{1}{2}$ of the total beans, so 6 beans must be black-eyed peas. Then I shared 12 beans in 4 groups because I need to find $\frac{1}{4}$ of the total beans. There were 3 beans in each group. I know that 3 beans are $\frac{1}{4}$ of the beans, so 3 beans must be pinto beans. The rest of the beans are red beans. If 6 are black-eyed peas and 3 are pinto beans, 6 + 3 = 9 and 12 − 9 = 3, so there are 3 beans left and those are red. The answer to riddle #3 is a set of 12 beans, 6 black-eyed peas, 3 pinto beans, and 3 red beans."

ESSENTIAL QUESTIONS

- How can I be sure fractional parts are equal in size?
- What do the numbers (terms) in a fraction represent?
- How does the number of equal pieces affect the fraction name?
- How can I write a fraction to represent a part of a group?
- How can I represent a fraction of a discrete model (a set)?
- How are multiplication, division, and fractions related?

MATERIALS

- *Clean-Sweep Campers* (2000) by Lucille Recht Penner, or similar book about fractions of a set

Each student will need the following materials:
- "A Bowl of Beans" student recording sheet
- Handful of mixed beans (red kidney beans, pinto beans, black beans, black-eyed peas, etc.)
- Small paper plates
- Colored pencils or crayons

GROUPING

Individual Task

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

Students write and solve riddles regarding fractions of a set. Students write number sentences using whole numbers and fractions.

Comments

To introduce this task read *Clean-Sweep Campers* (2000) by Lucille Recht Penner, illustrated by Paige Billin-Frye (A Math Matters book) or a similar book that discusses fractions of a set. Discuss the different solutions the girls devised for cleaning groups and/or discuss possible ways students could be grouped for Sports Day (see page 32).

Other related books that could be used with this task are *Jump, Kangaroo, Jump!: Fractions* (1999) by Stuart J Murphy, illustrated by Kevin O'Malley and *The Wishing Club: A story about fractions* (2007) by Donna Jo Naoli, illustrated by Anna Currey.

It is very important that beans are available for students to use. Students can use any type of bean initially, using the total number of beans given in the riddle. Once they find the fraction of the whole, they can trade the correct number of beans for the correct kind of bean.

After reviewing student-created riddles for accuracy, allow students to write their riddle on an index card with the solution drawn and explained on the back. Have students share their riddles with others in the class or leave them as an independent activity that is self-checking.

Task Directions

Students will follow the directions below from the "A Bowl of Beans" student recording sheet. You will need a bowl of different kinds of dried beans to solve the riddles below. Draw a picture of your set of beans for each riddle. Explain how you solved each riddle, using words and numbers. Finally, represent your answer with a number sentence (for example: if you have 6 beans and ⅓ are red then write the number sentence ⁶⁄₃ = 6 x ⅓ or 6 x ⅓ = ⁶⁄₃)

Riddle #1
- I have 8 beans.
- $\frac{1}{4}$ of my beans are red.
- The rest are pinto beans.

Show my set of beans.

Riddle #2
- $\frac{1}{3}$ of my beans are red.
- I have 9 beans.
- $\frac{2}{3}$ of my beans are black.

Show my set of beans.

Riddle #3
- $\frac{1}{2}$ of my beans are black-eyed peas.
- $\frac{1}{4}$ of my beans are pinto beans.
- The rest are red.
- I have 12 beans.

Show my set of beans.

Using beans create your own bean riddle below. Show the answer to your riddle on the back of this paper.

FORMATIVE ASSESSMENT QUESTIONS

- What information in the riddle did you need first?
- Is there any missing information in the riddle?
- If you have ___ (i.e., 9) beans in ___ (i.e., 3) equal groups, how many beans are in each group? How many groups do you need? How do you know?
- If we know $\frac{1}{3}$ of the beans are 3 beans, how many beans would be $\frac{2}{3}$ of the beans? How do you know?
- How did you know how many ___ (i.e., red) beans to use?
- How would this set of beans look if there were 12 beans instead of 9 total beans?

DIFFERENTIATION

Extension
- Ask students to create riddles with at least three types of beans.
- Have students explore equivalent fractions represented in a set (i.e., in a set of 9 beans if 1/3 of the set is red, then 3/9 of the beans are red).

Intervention
- An alternative activity could be used as follows:
  Riddle #1
  I have 6 beans. One-third $\left(\frac{1}{3}\right)$ of the beans are pinto beans. How many beans are pinto beans?
  Riddle #2
  I have 8 beans. One-eighth $\left(\frac{1}{8}\right)$ of the beans are black beans. How many beans are black beans?
  Riddle #3
  I have 12 beans. One-fourth $\left(\frac{1}{4}\right)$ of the beans are red beans. How many beans are red beans?

Name _____ Date _____

A Bowl of Beans

You will need a bowl of different kinds of dried beans to solve the riddles below. Draw a picture of your set of beans for each riddle. Explain how you solved each riddle using words and numbers. Finally, represent your answer with a number sentence (for example: if you have 6 beans and ⅓ are red then write the number sentence ⅔ = 6 x ⅓ or 6 x ⅓ = ⅔)

Riddle #1
- I have 8 beans.
- $\frac{1}{4}$ of my beans are red.
- The rest are pinto beans.

Show my set of beans.

Riddle #2
- $\frac{1}{3}$ of my beans are red.
- I have 9 beans.
- $\frac{2}{3}$ of my beans are black.

Show my set of beans.

Riddle #3
- $\frac{1}{2}$ of my beans are black-eyed peas.
- $\frac{1}{4}$ of my beans are pinto beans.
- The rest are red.
- I have 12 beans.

Show my set of beans.

Create your own bean riddle below. Show the answer to your riddle on the back of this paper.

Constructing Task: Birthday Cake!

STANDARDS FOR MATHEMATICAL CONTENT

MCC4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
   a. Understand a fraction *a/b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*
   b. Understand a multiple of *a/b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*
   c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

   For this activity students will be asked to determine the given number of candles on each piece of birthday cake when given a total number of candles on the cake. All problems assume that the cake pieces will be equal and everyone will always receive the same number of candles. The first part of the assignment provides students with whole to part problems. In other words, the students receive the whole amount in the question but need to produce the part of the whole to determine their answer. In the second part of the task students are given the number of candles on just one piece of cake or one fraction of the cake and have to then determine how many candles were on the entire cake. The second part of the assignment provides students with part to whole problems where they receive a part or fraction in the question but need to produce the whole amount to determine the answer. For example
   Before asking students to work on this task, be sure students are able to:
   • Use repeated addition to add fractions with the same denominator.

- Be able to decompose fraction, for example ⁴⁄₄ = ½ + ½ or ¼ + ¾.
- Have a strong understanding that the whole can be any number/size and the fractions always depend on taking a portion of this whole.

Whole to Part

The four people at Tanya's birthday will get one-quarter (one-fourth) of the cake each. Tanya puts 12 candles on the cake so that each person gets the same number of candles on their piece of cake. How many candles will each person get on their piece of cake?
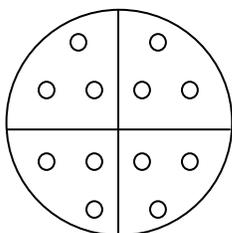
In the problem above the students need to determine the "part" when given the whole of 12 candles (i.e. 3 candles)

Part to Whole

Ricardo put enough candles on his birthday cake so that everyone would have the same number of candles. He then cut the cake into fourths. If each slice has three candles, how many candles did Ricardo put on his cake?
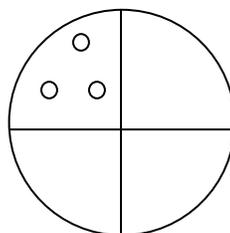
In the problem above the students need to determine the "whole" when given only the part (i.e. ¼ of the cake had 3 candles, therefore the whole must be 12 candles). Although not always the case, these types of problems often pose a greater challenge to 4th graders.

Many of these could be completed by using simple division or multiplication without fractions at all. However, students need to understand that the / or slash in any fraction really means the division operation. By completing tasks such as these students will begin to see a pattern and develop different strategies for multiplying and dividing denominators in order to solve problems that involve fractions.

ESSENTIAL QUESTIONS

- What does it mean to take a fraction portion of a whole number?
- How is multiplication of fractions similar to division of whole numbers?
- How do we determine the whole amount when given a fractional value of the whole?
- How do we determine a fractional value when given the whole number?

MATERIALS

- Paper plates or large circles either cut out or drawn
- Two sided counters, base ten units, or some other small counter
- Birthday Cake student recording sheet.

GROUPING

Group/Partner Task

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students will use a pie model to multiply a whole number by a fraction. Students will gain experience solving both part-to-whole and whole-to-part word problems that ask them to multiply a fraction by a whole number or multiply a whole number by a fraction

Comments

Paper plates and counters should be made available for students to act out each of these problems. If paper plates are not available, a large circle drawn on an 11 x 8.5 inch paper will work just as well. Students could also color or draw their candles, or use glue and die cuts.

This task could be introduced by bringing in a cake and showing students how to distribute the candles in such a way that everyone receiving cake would get the same number of candles.

Task Directions

Students will follow directions below from the Birthday Cake! task sheet.
- Obtain a set of counters and paper plates.
- Work with a partner or small group to make a fraction cake and record it on your task sheet.
- Be ready to articulate your reasoning.

FORMATIVE ASSESSMENT QUESTIONS

- How do you know how many pieces of cake there are?
- Can you write an equivalent fraction for your answer? (for the example above, ¼ = ³⁄₁₂)
- Are the candles evenly distributed or fairly distributed?
- What other situations do we need to share evenly?

DIFFERENTIATION

Extension
- Once students have completed the task above, this lesson could be extended to use larger numbers of candles and larger fractions.

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- Students could solve problems where the numerator is a number other than 1. For example, ⅚ of 30.

Intervention
- Students may need to review division and have experiences where they can see the relationship between the two.
- Students may use repeated addition to solve these problems.
- Students may be given cakes already "cut" or drawn in parts to help them realize what the denominator will be.

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Name _____ Date _____

Birthday Cake!
Part 1

-Act out the problem using circles and counters.
-Draw your answer using the circle.
-Explain your answer using words.
-Lastly, write a number sentence for each problem

1.   The four people at Carla's birthday will get one-quarter (one-fourth) of the cake each. Carla puts 16 candles on the cake so that each person gets the same number of candles on their piece of cake. How many candles will each person get on their piece of cake?
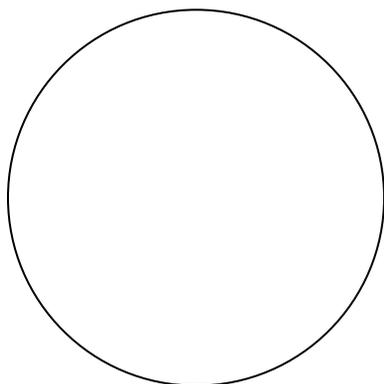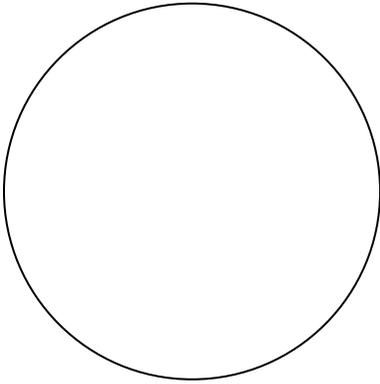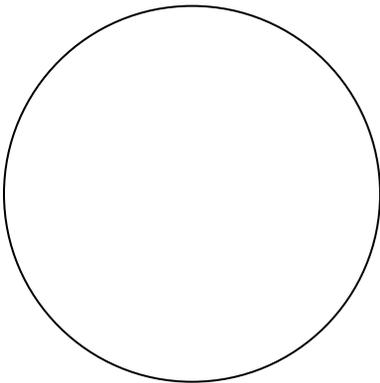
Explanation and Number Sentence

_____

_____

_____

_____

2.   The five people at Estella's birthday will get one-fifth of the cake each. Estella puts 25 candles on the cake so that each person gets the same number of candles on their piece of cake. How many candles will each person get on their piece of cake?
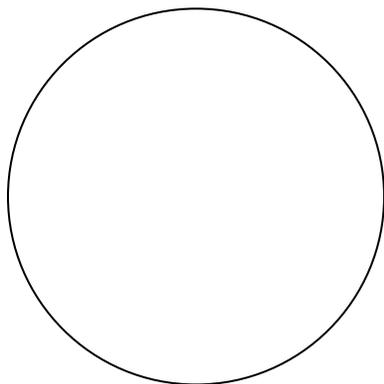
Explanation and Number Sentence

_____

_____

_____

_____

3.   Three people are at Emmanuel's birthday party. Emmanuel puts 21 candles on the cake and cuts it into thirds so that each person gets the same number of candles on their piece of cake. How many candles will each person get on their piece of cake?
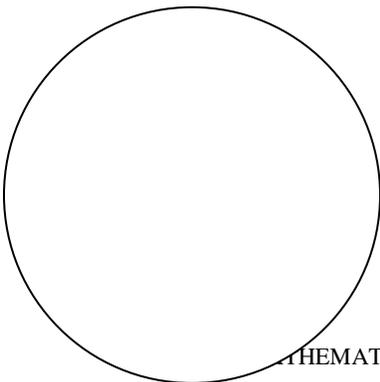
Explanation and Number Sentence

_____

_____

_____

_____

4.   The six people at Zoe's birthday will get one-sixth of the cake each. Zoe puts 18 candles on the cake so that each person gets the same number of candles on their piece of cake. However, one friend doesn't like cake so Zoe ate a second piece. How many candles will each person get on their piece of cake? How many candles will Zoe get?

Explanation and Number Sentence

_____

_____

_____

_____

5.   At the party, the cake is cut into quarters (fourths). Twelve candles are put on the cake. Greedy Greg eats three-quarters of the cake. How many candles does he get?
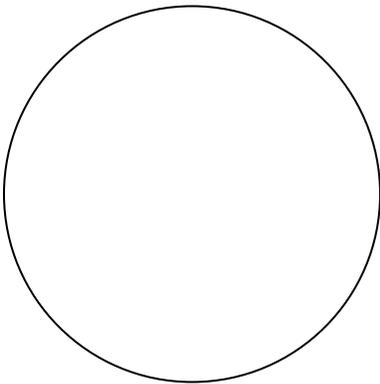
Explanation and Number Sentence

_____

_____

_____

_____

Name _____ Date _____

Birthday Cake!
Part 2

-Act out the problem using circles and counters.
-Draw your answer using the circle.
-Explain your answer using words.
-Lastly, write a number sentence for each problem.

1.   Reyna put enough candles on her birthday cake so that everyone would have the same number of candles. She then cut the cake into fifths. If each slice has four candles, how many candles did Reyna put on her cake?
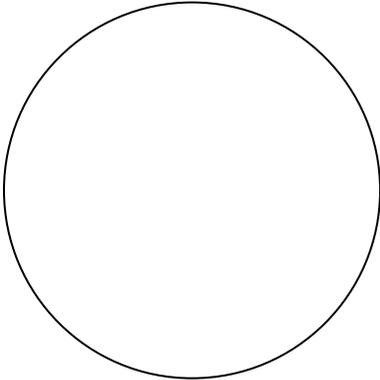
Explanation and Number Sentence

_____

_____

_____

_____

2.   Stan put enough candles on his birthday cake so that everyone would have the same number of candles. He then cut the cake into fourths. If each slice has six candles, how many candles did Stan put on his cake?
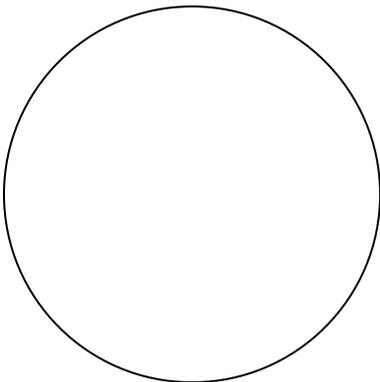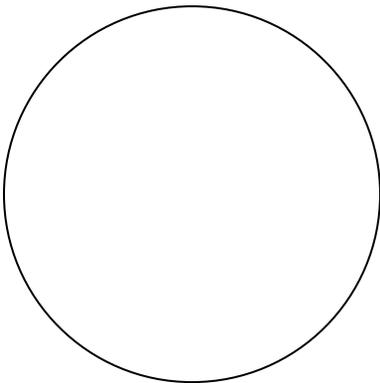
Explanation and Number Sentence

_____

_____

_____

_____

3.   Pedro put enough candles on his birthday cake so that everyone would have the same number of candles. After cutting himself a large slice he noticed that two-thirds of the cake has eight candles on it. How many candles are on the whole cake?

Explanation and Number Sentence

_____

_____

_____

_____

4.   Priya and her father made a cake for her birthday and put enough candles on it so that everyone would have the same number of candles. Priya's father cut the cake into fourths and gave Priya the first slice. He then noticed that the three-fourths of the cake that was left had twelve candles on it. How many candles were on the whole cake?
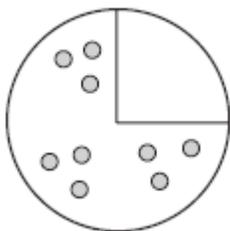
Explanation and Number Sentence
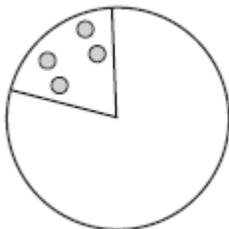
_____

_____

_____

_____

The cake below was cut into thirds. How many candles were on the whole cake?

_____
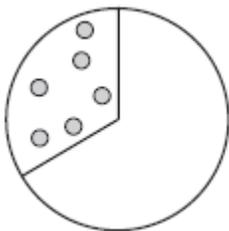
The cake below was cut into fourths. How many candles were on the whole cake?

_____

The cake below was cut into fifths. How many candles were on the whole cake?

Constructing Task: Fraction Clues

STANDARDS FOR MATHEMATICAL CONTENT

MCC4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
   a. Understand a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*
   b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*
   c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

   Students need practice with open-ended activities that allow them to design their own problems and then assess one another. This activity also makes students use mathematical language, verify answers, and work collaboratively with another student. This activity offers students a concrete way to see equivalent fractions.   This activity also helps build the "guess and check" strategy as each student tries to build the fraction bar based on the set of clues.

This activity is also valuable because students start to realize that a different number of tiles in a different fraction bar can still be represented by the same fraction. For example

3 tiles   4 tiles
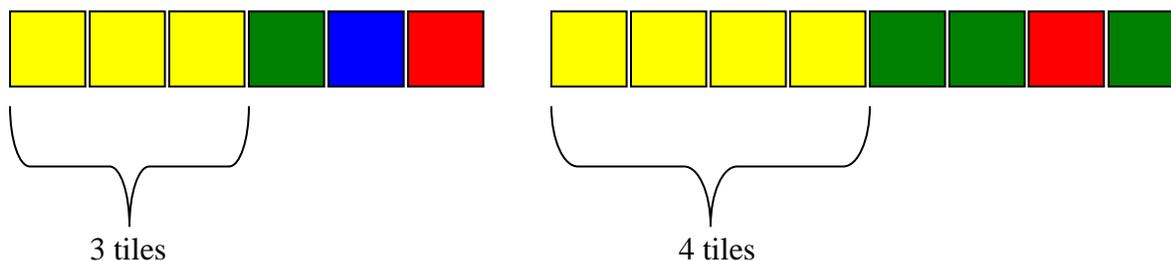
In the first bar three yellow tiles represent ½ and in the second bar four tiles represent ½. Students will gain further understanding that the number of tiles being used (numerator) is always dependent on its relationship to the total number of tiles (denominator)
Before asking students to work on this task, be sure students are able to:

- identify the number of equal pieces needed to cover one whole as the denominator
- show equivalent fractions with an area model
- record on the student sheet equivalent fractions or fraction sets (either by coloring or gluing die cut squares)
- write an equation which shows the clues and verify their answer.

ESSENTIAL QUESTIONS

- How can fraction represent parts of a set?
- How can I represent fractions in different ways?
- How can I find equivalent fractions?
- How can I multiply a set by a fraction?

MATERIALS

- Colored tiles
- Fraction Clues recording sheet
- Crayons or colored pencils

GROUPING

Individual/Partner Task

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task students will use what they have learned about adding and subtracting fractions, using equivalent fractions and multiplying a fraction by a whole number to give another student

clues about the fraction strip they created. There is a lot of emphasis on communicating mathematically in this task.

Comments

To introduce this activity display these two fraction bars made from Color Tiles.

Ask students to find out what portion of the whole a tile in the first bar represents and what portion of the whole a tile in the second bar represents. Students should be able to determine that each tile in the first bar represents ¼ of the whole and each tile in the second bar represents ⅙ of the whole.

Ask students to explain what fractional part each color represents in each fraction bar. Give the following set of fraction clues that describe one of the fraction bars. Stop after each clue and ask children which fraction bar is the solution and how they know.

- The fraction bar is one-half green
- The fraction bar is one-third red
- The fraction is one-sixth blue

Many children will not need all three clues to determine the solution however they should be comfortable arguing and verifying their answers and they may need all three clues to conclude that the solution is the second bar.

Part 2 is a much more challenging version where students create fractions bar with any number of tiles, requiring students to use different denominators, such as 6, 8, 10, and 12. This allows students to develop other strategies for determining the denominator, for example a student may be forced to find a common denominator or they may figure out on their own that the largest denominator must refer to the total number of tiles.

If available, students can glue die-cut red, yellow, blue and green squares.

Task Directions
Students will follow directions below from the Fraction Clues activity sheet. .
- Obtain a set of colored tiles.
- Work with a partner to make a fraction bar and record it on their activity sheet.
- Write at least 3 clues that describe your fraction bar
- Exchange only your clues with another group
- Represent your answer with a number sentences (for example: if you have 10 tiles and ½ are red then write the number sentence $^{10}\!/_2 = 10 \times \frac{1}{2}$ which is 5 tiles)
- Attempt to build another group's fraction bar as they attempt to build yours.
- Discuss results with each other.

FORMATIVE ASSESSMENT QUESTIONS

- What clues did you write to describe your fraction bar?
- Have you found all of the possible equivalent fractions? How do you know?
- Were you able to build the fraction bar based on the clues? If not, why?
- Could you change any of your clues?
- What number sentence can describe the tiles in your bar?

DIFFERENTIATION

Extension
- Once students have completed the task above, this lesson can be extended to have two pairs of students combine their fraction bars to make a larger fraction bar, then continue the activity writing clues for another group to solve.
- Students could also be encourage to work with larger fraction bars as well as write more clues for determining those fraction bars. Most color tiles only have red, blue, green and yellow tiles, so the activity will never have more than four fractions to represent.

Intervention
- If necessary students could begin this activity with a smaller set, such as using only four tiles.
- If students are struggling, they could attempt with activity with only three colors instead of using all four colored tiles.
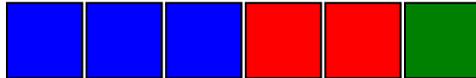
Name _____ Date _____

Fraction Clues
(Part 1)
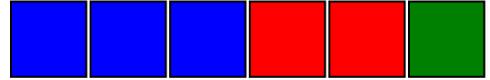
Make a Color Tile fraction bar and then write a set of clues so that someone else could build it.

• Work with a partner. Choose 6 Color Tiles and arrange them in any way to form a fraction bar.

• Decide what fractional part of the whole bar is represented by each color you used. For example:

Blue: ³⁄₆ or ½
Red: ²⁄₆ or ⅓
Green: ⅙

• Record your fraction bar on grid paper. Beneath the grid paper, write a several clues that describe the fractional parts of your bar. For example: *My bar is _____ blue.*

• Exchange lists with another pair. Be careful not to peek at the back of the list!
Follow the clues to try to build the other pair's fraction bar.

• Represent your answer with a number sentences (for example: if you have 10 tiles and ½ are red, then students might write the following: Half of 10 = ¹⁰⁄₂ = 10 ÷ ½ = 5 tiles)

• When you have finished making the fraction bar, turn the list of clues over and compare what you built to the recording.

• Discuss your results with the other pair.

Clue 1: _____

_____

_____

_____

Clue 2: _____

_____

_____

_____

Clue 3: _____

_____

_____

_____

Clue 4: _____

_____

_____

_____

• Work with a partner. Choose 8 color tiles and arrange them in any way to form a fraction bar.

• Decide what fractional part of the whole bar is represented by each color you used. For example:

Blue: ⅜
Red: ⅛ or 1/4
Green: ⅜

• Record your fraction bar on grid paper. Beneath the grid paper, write a several clues that describe the fractional parts of your bar. For example: *My bar is _____ blue.*

• Exchange lists with another pair. Be careful not to peek at the back of the list!
Follow the clues to try to build the other pair's fraction bar.

• Represent your answer with a number sentences (for example: if you have 10 tiles and ½ are red then write the number sentence ¹⁰½ = 10 ÷2 = 5 tiles)

• When you have finished making the fraction bar, turn the list of clues over and compare what you built to the recording.

• Discuss your results with the other pair.

Clue 1: _____

_____

_____

_____

Clue 2: _____

_____

_____

_____

Clue 3: _____

_____

_____

_____

Clue 4: _____

_____

_____

_____

• Work with a partner. Choose 12 Color Tiles and arrange them in any way to form a fraction bar.

• Decide what fractional part of the whole bar is represented by each color you used. For example:
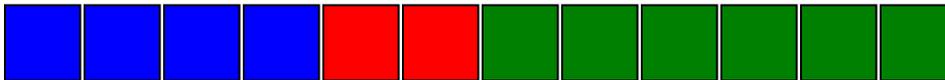
Blue: $\frac{4}{12}$ or $\frac{1}{3}$
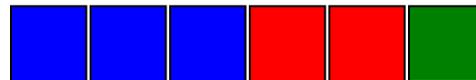Red: $\frac{2}{12}$ or $\frac{1}{6}$
Green: $\frac{6}{12}$ or $\frac{1}{2}$

• Record your fraction bar on grid paper. Beneath the grid paper, write a several clues that describe the fractional parts of your bar. For example: *My bar is _____ blue.*

• Exchange lists with another pair. Be careful not to peek at the back of the list!
Follow the clues to try to build the other pair's fraction bar.

• Represent your answer with a number sentences (for example: if you have 10 tiles and ½ are red then write the number sentence $\frac{1}{2}$ = 10 ÷2 = 5 tiles)

• When you have finished making the fraction bar, turn the list of clues over and compare what you built to the recording.

• Discuss your results with the other pair.

Clue 1: _____

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Clue 2: _____

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_____

Clue 3: _____

_____

_____

_____

Clue 4: _____

_____

_____

_____

Name _____ Date _____

Fraction Clues
(Part 2)

Make a Color Tile fraction bar and then write a set of clues so that someone else could build it

• Work with a partner. Choose any number of Color Tiles and arrange them in any way to form a fraction bar.

• Decide what fractional part of the whole bar is represented by each color you used. For example:

Blue: ⅜ or ½
Red: ⅔ or ⅓
Green: ⅙

• Record your fraction bar on grid paper. Beneath the grid paper, write a several clues that describe the fractional parts of your bar. For example: *My bar is _____ blue.*

• Exchange lists with another pair. Be careful not to peek at the back of the list!
Follow the clues to try to build the other pair's fraction bar.

• Represent your answer with a number sentences (for example: if you have 10 tiles and ½ are red then write the number sentence $^{10}\!/_2 = 10 \div 2 = 5$ tiles)

• When you have finished making the fraction bar, turn the list of clues over and compare what you built to the recording.
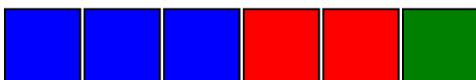
• Discuss your results with the other pair.

Clue 1: _____

_____

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Clue 2: _____

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_____

Clue 3: _____

_____

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_____

Clue 4: _____

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_____

Georgia Department of Education
Common Core Georgia Performance Standards Framework
Fourth Grade Mathematics • Unit 4

CONSTRUCTING TASK: Area Models

STANDARDS FOR MATHEMATICAL CONTENT

MCC4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
  a. Understand a fraction *a/b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*
  b. Understand a multiple of *a/b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*
  c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

   Students are often taught at younger ages that the operation of multiplication produces a larger product and the operation of division produces a smaller quotient. Multiplying whole numbers does produce larger numbers and dividing whole numbers does produce smaller numbers. However, when students begin using these operations with fractions the exact opposite occurs. This task will illustrate that idea. Below is one example of how the task could be accomplished.
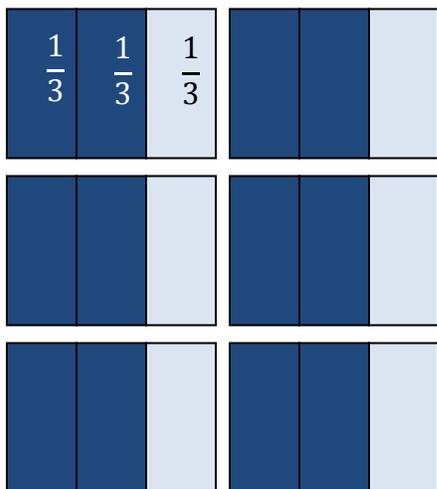
Draw an area model to represent each of the following operations. Use your area model to help you compute the answer to each problem.

$$6 \cdot \frac{2}{3}$$

Answers will vary.

Possible Solution

A possible solution for $6 \cdot \frac{2}{3}$ is below. This model shows six rectangles with each having $\frac{2}{3}$ of their area shaded. The results show $\frac{12}{3}$ shaded which is equivalent to 4 whole rectangles.

ESSENTIAL QUESTIONS

- What strategies can be used for finding products when multiplying a whole number by a fraction?
- How can I model the multiplication of a whole number by a fraction?

MATERIALS

- Colored pencils or crayons
- Area Model recording sheet

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

In this task, students will use area models to demonstrate their conceptual understanding of multiplying a whole number by a fraction. Teachers should support good student dialogue and take advantage of comments and questions to help guide students into correct mathematical thinking.

Comments

The area model of representing fraction is an important way for students to understand both multiplication and division of fractions. This task would allow students some freedom to create their own models and create a miniature book to represent the difference between multiplying a whole number by a fraction and dividing a whole number by another whole number.

Task Directions
      Have students follow the directions on the area model recording sheet.
Use the square below to draw an area model to represent the following multiplication problems.
Use your area model to help you compute the answer to each problem.

FORMATIVE ASSESSMENT QUESTIONS

- What did you notice when you multiplied a whole number by a fraction? Did this surprise you? Why or why not?
- How is multiplying a whole number by a fraction different than multiplying a whole number by another whole number?
- How is dividing a whole number by another whole number similar to multiplying a whole number by a fraction?

DIFFERENTIATION

Extension
- Students can extend this activity by creating other models, such as a set model for multiplying a whole number by a fraction.

Intervention
- Students could begin by multiplying whole numbers with fractions that have a numerator of 1, such as ¼, ½, ⅓, before moving on to fractions with larger numerators.

Name _____ Date _____

Area Models: Multiplication

Use the square below to draw an area model to represent the following multiplication problems. Use your area model to help you compute the answer to each problem.

6 · $\dfrac{2}{3}$

Explanation:

_____

_____

_____

_____

_____

_____

_____

_____

_____

$$8 \cdot \frac{3}{4}$$

Explanation:

_____

_____

_____

_____

_____

_____

_____

_____

_____

$\dfrac{2}{5} \cdot \mathbf{6}$

Explanation:

_____

_____

_____

_____

_____

_____

_____

_____

Use the square below to draw an area model to represent the following operations. Use your area model to help you compute the answer to each problem. What happens if you switch the equation around to read ½ times 8?

$$8 \cdot \frac{1}{2}$$

Practice Task: Fraction Pie Game

STANDARDS OF MATHEMATICAL CONTENT

MCC4.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
a. Understand a fraction *a/b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*
b. Understand a multiple of *a/b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

When asked what is larger, ½ or ¼, many students will say that ¼ is larger. When asked why, they will say, "Because four is a larger number than two." Students need a lot of exposure to fractions to discover various essential fractional concepts. One important concept involves students discovering the relationship between the size of a fractional piece and the size of the denominator. The greater the denominator, the smaller the fraction. This game is intentionally broken up into two parts so students can see that as the denominator gets higher. it in fact becomes much more difficult and requires many more roles of the dice, to produce 15 wholes. Young students also typically enjoy games of chance. and probability is a skill that also builds fractional understanding. Having students play with both 6-sided and 10-sided dice will also build this skill and they will begin to gain some understanding of both fractions and their odds when playing games of chance.
   Before asking students to work on this task, be sure students are able to:
   - Use repeated addition to add fractions with the same denominator.
   - decompose fractions, for example ⁴⁄₄ = ½ + ½ or ¼ + ¾
   - understand that the whole came be any number and the fractions always depend on taking a portion of this whole

ESSENTIAL QUESTIONS

- What does it mean to take a fractional portion of a whole number?
- How is multiplication of fractions similar to division of whole numbers?
- How is multiplication of fractions similar to repeated addition of fraction?
- What is the relationship between the size of the denominator and the size of each fractional piece (i.e. the numerator)?

MATERIALS

- Fraction circles
- Colored pencils or crayons
- 6 or 10 sided dice
- Fraction Pie Game student recording sheet.

GROUPING

Group/Partner Task

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task students will play a simple game of chance to see who can fill up 15 wholes on their score card and game board first. Children at this age enjoy games of chance, however this game also gives students a chance to practice using their fractional understandings and also requires them to use logical thinking and problem solving strategies.

Comments

Colored fraction circles should be made available to the students as much as possible. However, if these manipulatives are not available, then student's score cards must also be colored in. Fraction bars can be used as an alternative manipulative for this task. Students may even utilize the fraction bars as a strategy for keeping track of their score.

This task could be introduced by playing similar commercially bought dice games, such as Yahtzee, which are great, but often reinforce whole number operations and not fractions. Some students might be familiar with these games. To teach the students how to play the game, the teacher could model with a student and play one round. To play the game, a player rolls the dice. The value of the roll indicates the number of fraction pieces you can shade in.

The game is designed to build some fractional thinking alongside some strategy. For example, if a player rolled a 6, they could shade in a fraction with a larger denominator such as tenths or twelfths. Students will quickly realize that the greater the denominator, the harder it is to cover up that fraction.

Task Directions
   Students will follow directions below from the Fraction Pie Game recording sheet.
 The object of the game is to be the first person to 15 wholes. You have to complete a whole for it count toward the 15. You need to be able to prove your answer.

2-4 person game

1. Each person takes turns rolling the dice
2. After you have rolled, you must pick what type of pie you choose to color in and then you may color in fractional pie for the value of the role (for example, if you rolled a 6 and chose halves then you may shade in 6 halves)
3. You are also responsible for filling in your score card as you play.
4. Play several rounds, but be sure to share your strategy when you are done.
5. Be sure you can prove your answer.

FORMATIVE ASSESSMENT QUESTIONS

- What fraction circles are you trying to fill up first? Why?
- What strategies do you have for this game?
- If you roll a high number, what fraction circle might you try to fill in?
- If you roll a low number, what fraction circle might you try to fill in?

DIFFERENTIATION

Extension
- Once students have completed the task above, this lesson could be extended to use a larger value then 15, although more score cards will need to be reproduced. For example, if two score cards were made available, students could play to 20 or 30.
- Students could also play with additional dice.
- Students could be introduced to much larger fractions such as ½₅, ½₀, ½₀₀.

Intervention
- Students may need to play to a smaller whole than 15.
- Students could benefit from a number line such as the one included below.

1											
$\frac{1}{2}$						$\frac{2}{2}$					
$\frac{1}{3}$				$\frac{2}{3}$				$\frac{3}{3}$			
$\frac{1}{4}$			$\frac{2}{4}$			$\frac{3}{4}$			$\frac{4}{4}$		
$\frac{1}{5}$		$\frac{2}{5}$		$\frac{3}{5}$		$\frac{4}{5}$		$\frac{5}{5}$			
$\frac{1}{6}$		$\frac{2}{6}$		$\frac{3}{6}$		$\frac{4}{6}$		$\frac{5}{6}$		$\frac{6}{6}$	
$\frac{1}{7}$	$\frac{2}{7}$		$\frac{3}{7}$		$\frac{4}{7}$		$\frac{5}{7}$		$\frac{6}{7}$		$\frac{7}{7}$
$\frac{1}{8}$	$\frac{2}{8}$		$\frac{3}{8}$		$\frac{4}{8}$		$\frac{5}{8}$		$\frac{6}{8}$	$\frac{7}{8}$	$\frac{8}{8}$
$\frac{1}{9}$	$\frac{2}{9}$		$\frac{3}{9}$		$\frac{4}{9}$	$\frac{5}{9}$		$\frac{6}{9}$	$\frac{7}{9}$	$\frac{8}{9}$	$\frac{9}{9}$
$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	$\frac{10}{10}$		
$\frac{1}{11}$	$\frac{2}{11}$	$\frac{3}{11}$	$\frac{4}{11}$	$\frac{5}{11}$	$\frac{6}{11}$	$\frac{7}{11}$	$\frac{8}{11}$	$\frac{9}{11}$	$\frac{10}{11}$	$\frac{11}{11}$	
$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{5}{12}$	$\frac{6}{12}$	$\frac{7}{12}$	$\frac{8}{12}$	$\frac{9}{12}$	$\frac{10}{12}$	$\frac{11}{12}$	$\frac{12}{12}$

1											
$\frac{1}{2}$						$\frac{2}{2}$					
$\frac{1}{3}$				$\frac{2}{3}$				$\frac{3}{3}$			
$\frac{1}{4}$			$\frac{2}{4}$			$\frac{3}{4}$			$\frac{4}{4}$		
$\frac{1}{5}$		$\frac{2}{5}$		$\frac{3}{5}$		$\frac{4}{5}$		$\frac{5}{5}$			
$\frac{1}{6}$		$\frac{2}{6}$		$\frac{3}{6}$		$\frac{4}{6}$		$\frac{5}{6}$		$\frac{6}{6}$	
$\frac{1}{7}$	$\frac{2}{7}$		$\frac{3}{7}$		$\frac{4}{7}$		$\frac{5}{7}$		$\frac{6}{7}$		$\frac{7}{7}$
$\frac{1}{8}$	$\frac{2}{8}$		$\frac{3}{8}$		$\frac{4}{8}$		$\frac{5}{8}$		$\frac{6}{8}$	$\frac{7}{8}$	$\frac{8}{8}$
$\frac{1}{9}$	$\frac{2}{9}$		$\frac{3}{9}$		$\frac{4}{9}$	$\frac{5}{9}$		$\frac{6}{9}$	$\frac{7}{9}$	$\frac{8}{9}$	$\frac{9}{9}$
$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	$\frac{10}{10}$		
$\frac{1}{11}$	$\frac{2}{11}$	$\frac{3}{11}$	$\frac{4}{11}$	$\frac{5}{11}$	$\frac{6}{11}$	$\frac{7}{11}$	$\frac{8}{11}$	$\frac{9}{11}$	$\frac{10}{11}$	$\frac{11}{11}$	
$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{5}{12}$	$\frac{6}{12}$	$\frac{7}{12}$	$\frac{8}{12}$	$\frac{9}{12}$	$\frac{10}{12}$	$\frac{11}{12}$	$\frac{12}{12}$

Name: _____   Date: _____

Fraction Pie Game
Part 1
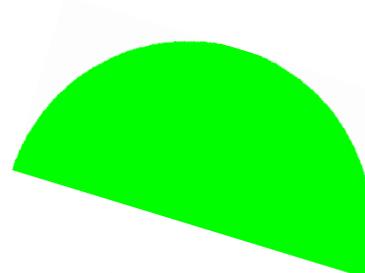
The object of the game is to be the first person to make 15 wholes. You have to complete a whole for it count toward the 15. You need to be able to prove your answer.
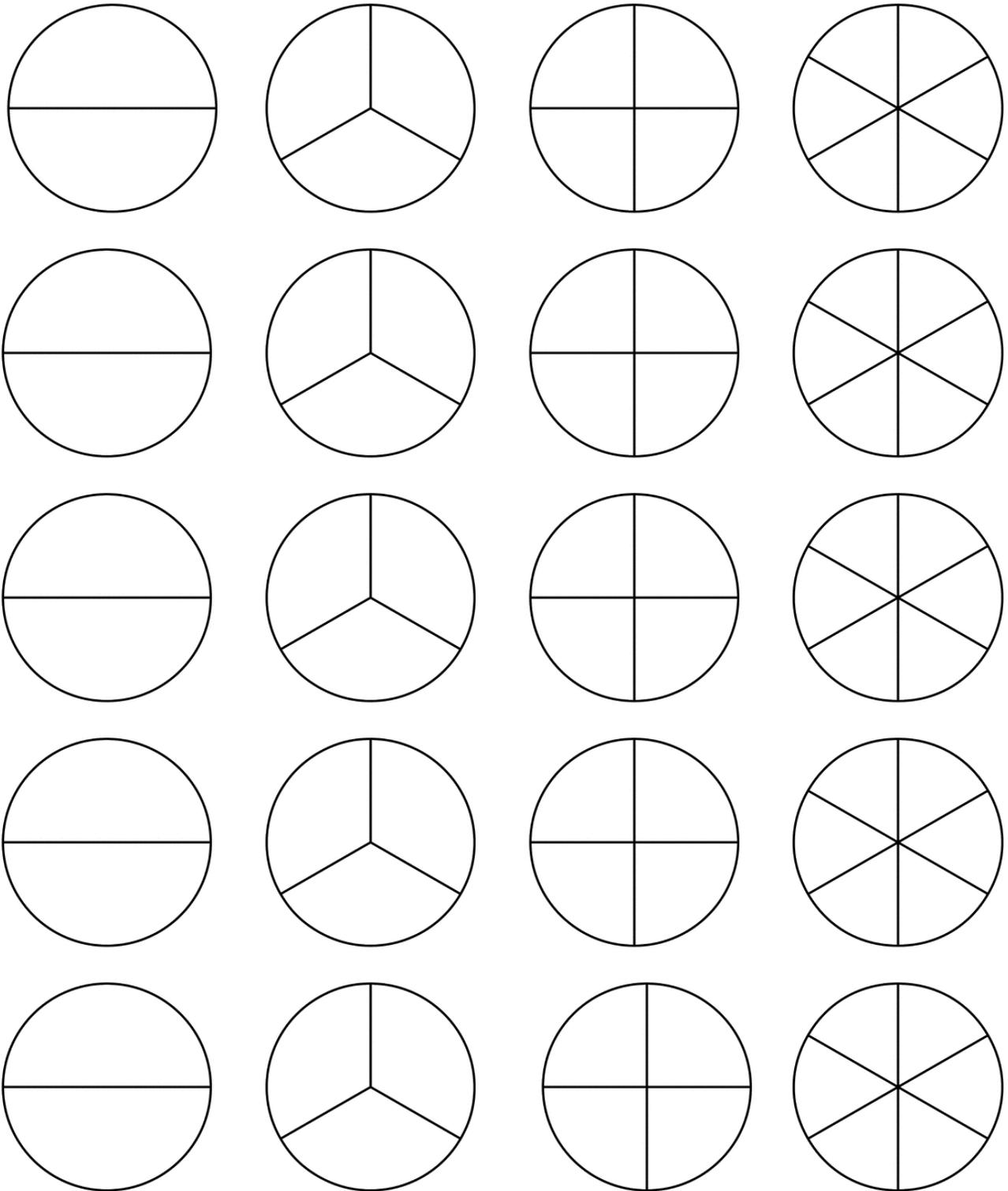
2-4 person game

1. Each person takes turns rolling the dice
2. After you have rolled, you must pick what type of pie you choose to color in and then you may color in fractional pie for the value of the roll (for example, if you rolled a 6 and chose halves then you may shade in 6 halves)
3. You are also responsible for filling in your score card as you play.
4. Play several rounds, but be sure to share your strategy when you are done.
5. Be sure you can prove your answer.

Fraction	Tally	Fraction x Tally	Product
½			
⅓			
¼			
⅙			

TOTAL: _____

_____

_____

_____

Name: _____   Date: _____

Fraction Pie Game
Part 2

The object of the game is to be the first person to 15 wholes. You have to complete a whole for it count toward the 15. You need to be able to prove your answer.

2-4 person game

1. Each person takes turns rolling the dice
2. After you have rolled, you must pick what type of pie you choose to color in, and then you may color in fractional pie for the value of the roll (for example, if you rolled a 6 and chose halves, then you may shade in 6 halves)
3. You are also responsible for filling in your score card as you play.
4. Play several rounds, but be sure to share your strategy when you are done.
5. Be sure you can prove your answer.

Fraction	Tally	Fraction x Tally	Product
$\frac{1}{5}$			
$\frac{1}{8}$			
$\frac{1}{10}$			
$\frac{1}{12}$			

TOTAL: _____

_____

_____

_____

Was it easier to get to 15 in Part 1 or Part 2? Why do you suppose this is? _____

_____

MATHEMATICS • GRADE 4• UNIT 4: Multiplying and Dividing Fractions
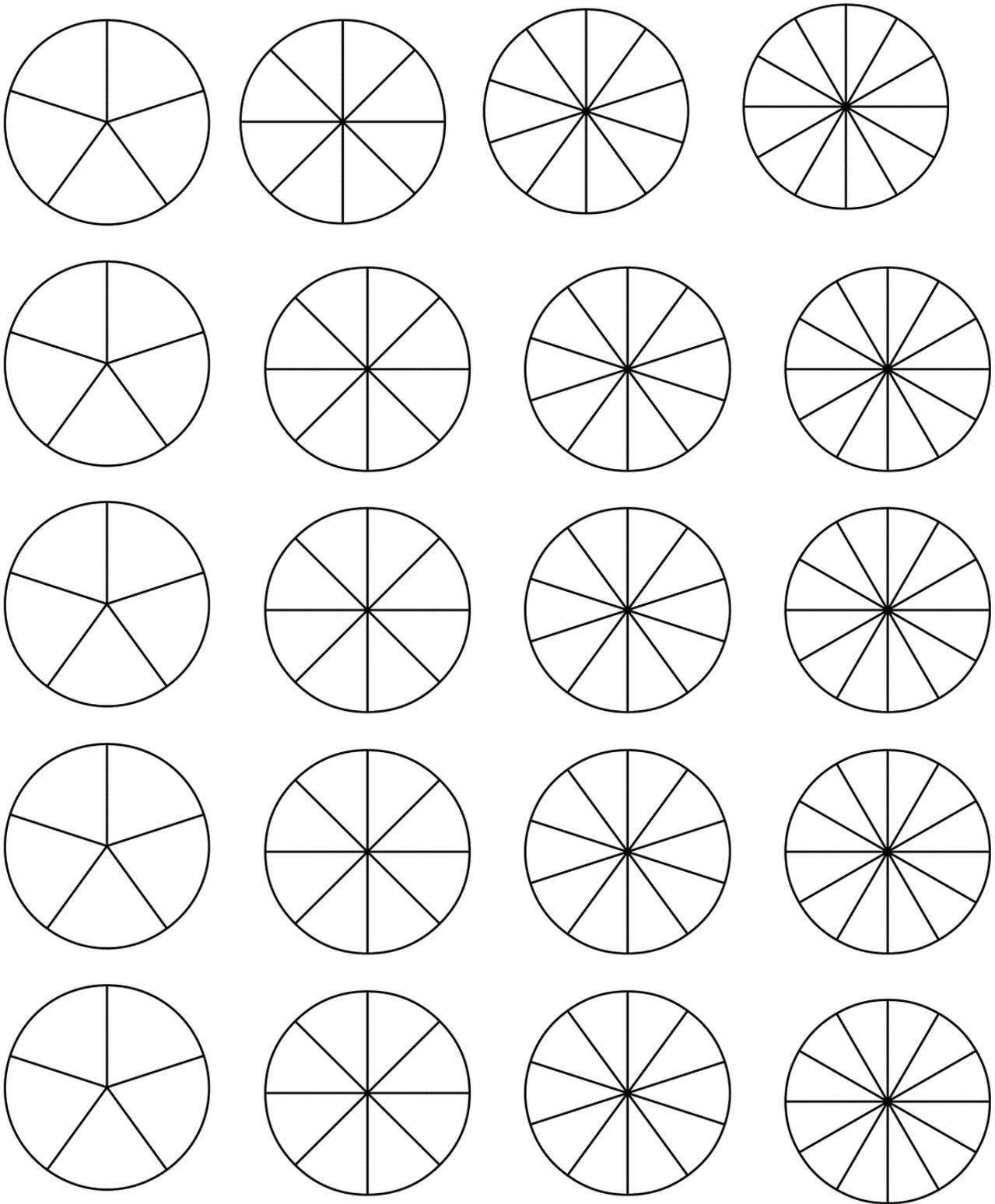
Constructing Task: Birthday Cookout

STANDARDS FOR MATHEMATICAL CONTENT

MCC4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction *a/b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*

b. Understand a multiple of *a/b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

You may want to review problem solving strategies with your students as they begin work on this task. Strategies such as making a table and working backward are two approaches to this task. Another suggestion for solving fraction word problems such as this is to utilize the Singapore Math strategy of drawing bars that are proportionate to the values in the problem. For example, we know that 80 people ordered hamburgers so we can draw a large bar to represent the hamburgers. We can then draw a bar ½ the size of our "hamburger" bar to represent the number of people that want hotdogs. Next we can draw a bar that is ¼ the size of our "hot dog" bar to represent the number of people that want steak. Finally, we can draw a bar ⅕ the size of our "steak" bar to represent the number of people that want chicken.

Chicken (2)

| Steaks (10) | | | |

Hot Dogs (40)

Hamburgers (80)
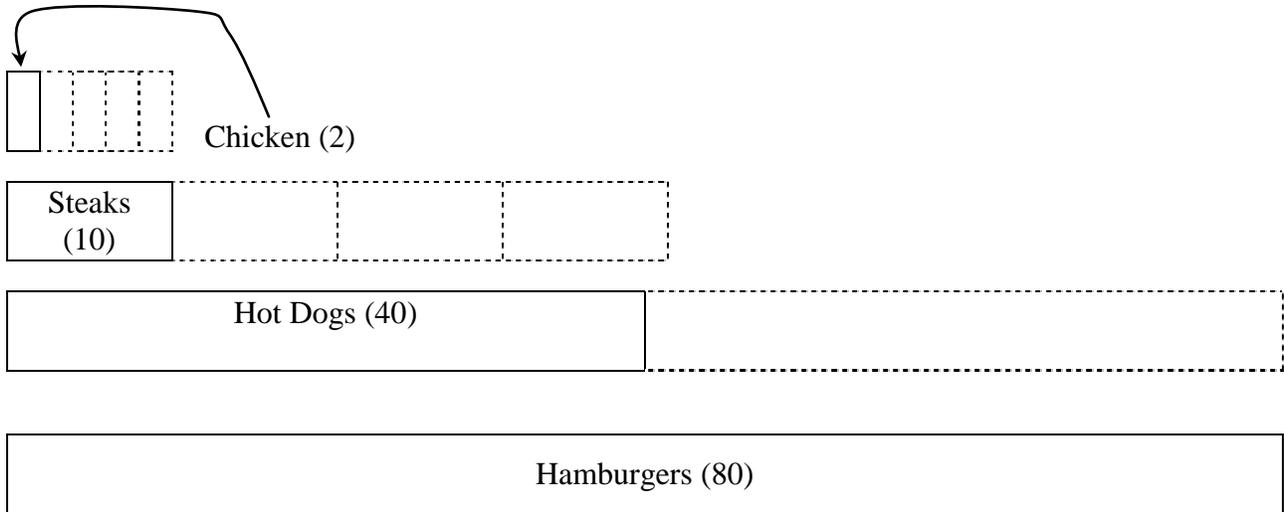
ESSENTIAL QUESTIONS

- How can we use fractions to help us solve problems?
- How can we model answers to fraction problems?
- How can we write equations to represent our answers when solving word problems?

MATERIALS

- "Birthday Cookout" student recording sheet

GROUPING

Partner/Small Group Task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

This task asks students to use problem solving strategies and their knowledge of fractions to solve a real-world problem involving food for a birthday party.

Comments

The setting of this task is likely a familiar one for students. You may want to begin with a discussion of how math is used when planning a birthday party. The discussion may include a wide range of mathematical ideas such as number of invitations, amount of food, and the amount of money needed to purchase food.

Solutions are given below:

- How many people asked for chicken? (1/5 of 10 is 2)
- How many people asked for steak? (1/4 of 40 is 10)
- How many asked for hot-dogs? (1/2 of 80 is 40)

Task Directions
   Have students follow the directions on the "Birthday Cookout" student recording sheet.

FORMATIVE ASSESSMENT QUESTIONS

- What problem solving strategies will you use to solve this problem?
- What models will you use to determine what the chef needs to know?
- How are you using fractions to help solve this problem?

DIFFERENTIATION

Extension
- Have students research and determine the cost of the items the chef needs.
- Have students create their own menu and create a new problem involving fractions.
- Have students determine the percentage of guests who chose each menu item.

Intervention
- Use smaller numbers, for example instead of 80 hamburgers, use 40 hamburgers.
  - How many people asked for chicken? (1/5 of 5 is 1)
  - How many people asked for steak? (1/4 of 20 is 5)
  - How many asked for hot-dogs? (1/2 of 40 is 20)

Name _____ Date _____

Birthday Cookout

Bob turned 60 this year! His family celebrated by having a cookout. Marcy took orders and found one fifth as many people wanted chicken as wanted steaks, one fourth as many people wanted steaks as wanted hot dogs, and one half as many people wanted hot dogs as wanted hamburgers. She gave her son-in-law, the chef, an order for 80 hamburgers.

The chef needs more information. He has to know:

- How many people asked for chicken?
- How many people asked for steak?
- How many asked for hot-dogs?

Use words, pictures and numbers to tell the chef what he needs to know. Be prepared to share!

<u>PERFORMANCE TASK:</u> Fraction Farm

<u>STANDARDS FOR MATHEMATICAL CONTENT</u>

MCC4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
a. Understand a fraction *a/b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*
b. Understand a multiple of *a/b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

<u>STANDARDS FOR MATHEMATICAL PRACTICE</u>

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

<u>BACKGROUND KNOWLEDGE</u>

A great strategy for this type of problems solving task is to construct some sort of table. Students have been exposed to tables throughout this unit, however, the tasks have always included a table and students are then asked to fill it in. For this task, a table has not been included intentionally, with the hope the students will have learned the value of organizing their information.

For this task, students should be allowed to use pattern blocks because rearranging the pattern blocks will allow them to make four hexagons and therefore start to determine the fractional value of each piece. Once the fraction is determined, students then begin multiplying the total value of $1,200 by each fraction. For example, Field C is a hexagon and represents ¼ of the property and is therefore valued at ¼ x $1,200 = $300. If students are not allowed to manipulate pattern blocks they may struggle with finding each fractional value. Alternatively, students could be provided with isometric pattern block paper, draw the field, and determine the fractional value that way. Isometric pattern block paper has been included with this task.

Answer Key:

Field	Shape	Fraction of Whole	Dollar Value
T	*Trapezoid*	*⅛ or ³⁄₂₄*	*$150*
B	*Parallelogram*	*¹⁄₁₂ or ²⁄₂₄*	*$100*
C	*Hexagon*	*¼ or ⁶⁄₂₄*	*$300*
D	*Parallelogram*	*¹⁄₁₂ or ²⁄₂₄*	*$100*
E	*Triangle*	*¹⁄₂₄*	*$50*
F	*Parallelogram*	*¹⁄₁₂ or ²⁄₂₄*	*$100*
G	*Trapezoid*	*⅛ or ³⁄₂₄*	*$150*
H	*Trapezoid*	*⅛ or ³⁄₂₄*	*$150*
I	*Triangle*	*¹⁄₂₄*	*$50*
J	*Triangle*	*¹⁄₂₄*	*$50*
			TOTAL: $1,200

ESSENTIAL QUESTIONS

- How can I multiply a whole number by a fraction?
- What is the relationship between multiplication by a fraction and division?
- How can I represent multiplication of a whole number?

MATERIALS

- Pattern Blocks
- Fraction Farm student handout
- Crayons or colored pencils

GROUPING

Individual/Partner Task

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task students are asked to determine the value of the fractional pieces of a farm. The farm has been divided into several plots, each representing a fraction of the whole.

Task Directions
- The farm fields above are worth a total of $1,200. The fields are divided up like pattern blocks; hexagons, trapezoids, parallelograms and triangles. Each field's value is based on its size.
- What part of the whole farm does each field represent?

- What is the dollar value of each field?
- Show and explain all of your mathematical thinking.

FORMATIVE ASSESSMENT QUESTIONS

- How can pattern block or isometric pattern block graph paper help you determine each fraction?
- Is there another way to represent that fraction?
- How are you going to organize your work?
- What is the value of the hexagon and what shapes can cover a hexagon?

DIFFERENTIATION

Extension
- Once students have completed the task above, this lesson can be extended by changing the value of the land to $2,400 or $3,600.
- Students could also be encourage create their own field with subplots of land using pattern blocks. This would be an open ended assignment and require a higher degree of critical thinking, but very worthwhile. It would also self- differentiate. Students ready for a more complicated fractional field could build larger designs with lots of pattern blocks.
- Students could work in pairs to design their own farm, then trade their design with another pair of students and solve another's design.

Intervention
- If necessary, students could make a smaller design that is less complicated.
- Students struggling with multiplying a whole number by a fraction could begin with a smaller whole number such as $240.
- Some students may still need to be provided with a table. See below:

Field	Shape	Fraction of Whole	Dollar Value
T	Trapezoid	⅛ or ³⁄₂₄	$150
B	Parallelogram	¹⁄₁₂ or ²⁄₂₄	$100
C	Hexagon	¼ or ⁶⁄₂₄	$300
D	Parallelogram	¹⁄₁₂ or ²⁄₂₄	$100
E	Triangle	¹⁄₂₄	$50
F	Parallelogram	¹⁄₁₂ or ²⁄₂₄	$100
G	Trapezoid	⅛ or ³⁄₂₄	$150
H	Trapezoid	⅛ or ³⁄₂₄	$150
I	Triangle	¹⁄₂₄	$50
J	Triangle	¹⁄₂₄	$50
			TOTAL: $1,200

Name _____ Date _____

Fraction Farm
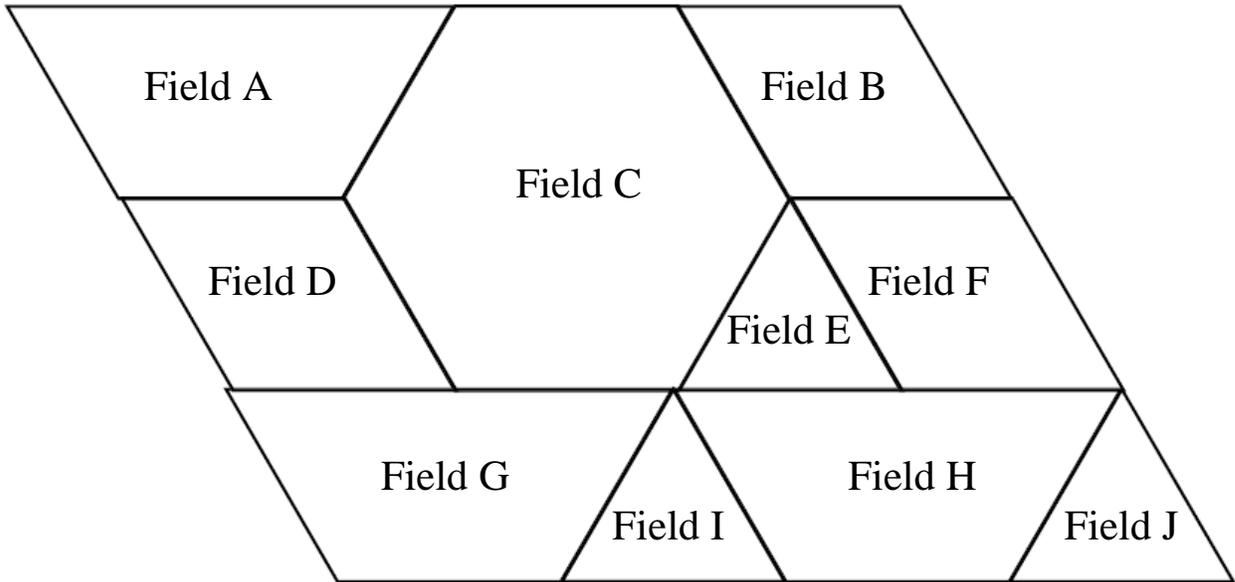
The farm fields above are worth a total of $1,200. The fields are divided up like pattern blocks; hexagons, trapezoids, parallelograms and triangles. Each field's value is based on its size (its fractional relationship the whole plot of farm land). What part of the whole farm does each field represent? What would a fair price for each field be, based on its fractional value? Show and explain all of your mathematical thinking.

## CCSS Mathematics Content Standards Rubric – Grades 3 & 4

Students apply mathematical concepts, reasoning, and procedural skills in problems-solving situations and support their solutions using computations, mathematical language, and appropriate representations/ modeling.

CCSS Math Criteria by Strand	Novice	Apprentice	Practitioner	Expert
Number & Operations in Base Ten , Number & Operations – Fractions, & Operations & Algebraic Thinking	Applies flawed strategies (e.g., attempts to form groups when multiplying, but does not use equal sized groups) Selects the incorrect operation to perform or major inaccuracies in computation lead to an incorrect solution Still demonstrates limited knowledge of place value or number sense (e.g., difficulty estimating, representing part-whole relationships; cannot determine reasonableness of an answer; does not see relationship between multiplication-division) A correct answer may be stated, but is not supported by student work or explanations	Some parts of problem correct and those parts are supported by student work Uses visual models (number line, area models, set models) to represent and compare parts of a whole, but stops short of applying concepts in problem solving contexts Demonstrates relationships between multiplication and division with whole numbers using number facts, objects, visuals, and symbols/equations May include limited/ partial explanations for solutions Uses 4 operations with small whole numbers, but displays some inaccuracies in computations of large numbers (multi-digit) and small numbers (fractions, decimals)	*Expresses whole numbers as fractions (gr 3) and fractions as decimals with 10 or 100 as denominator (gr 4)* 3.NF-3 4.NF-5, 6 *Uses place value to round whole numbers* 3.NBT-1 4.NBT-3 *Expresses fractions and equivalent fractions (gr 3-4) and decimal-fraction equivalents (gr 4); explains/ illustrates why they are/are not equivalent (e.g., number lines, area models, sets; compare to benchmarks)* 3.NF-1, 2, 3 4.NF-1, 2, 5, 6, 7 *Uses addition, subtraction, and multiplication to solve problems with fractions (gr3-4) and mixed numbers (gr 4)* 3.NF-1, 2, 3 4.NF-3, 4 *Uses 4 operations in solving multi-step problems and word problems with whole numbers (e.g., using equations, arrays, explaining patterns using whole numbers, following a rule)* 3.OA-3, 4, 5, 6, 7, 8, 9 3.NBT-2, 3 4.OA-2, 3, 5 *May be some minor flaws when performing multi-step computations, <u>but procedural and conceptual understanding is clearly evident</u>*	All parts of problem correct, precise, and supported by student work or explanations Extends understanding of equivalence of fractions by identifying proper and improper fractions Interprets meaning of the products when multiplying (gr 3-4) and remainders when dividing (gr 4) Uses a variety of representations (e.g., concrete models, diagrams, equations), strategies (e.g., place value, properties of operations), and algorithms to solve problems or represent solutions in multiple ways Uses additive or multiplicative reasoning to solve or interpret most problems

NOTE: Anchor papers will illustrate how descriptors for each performance level are evidenced at each grade.
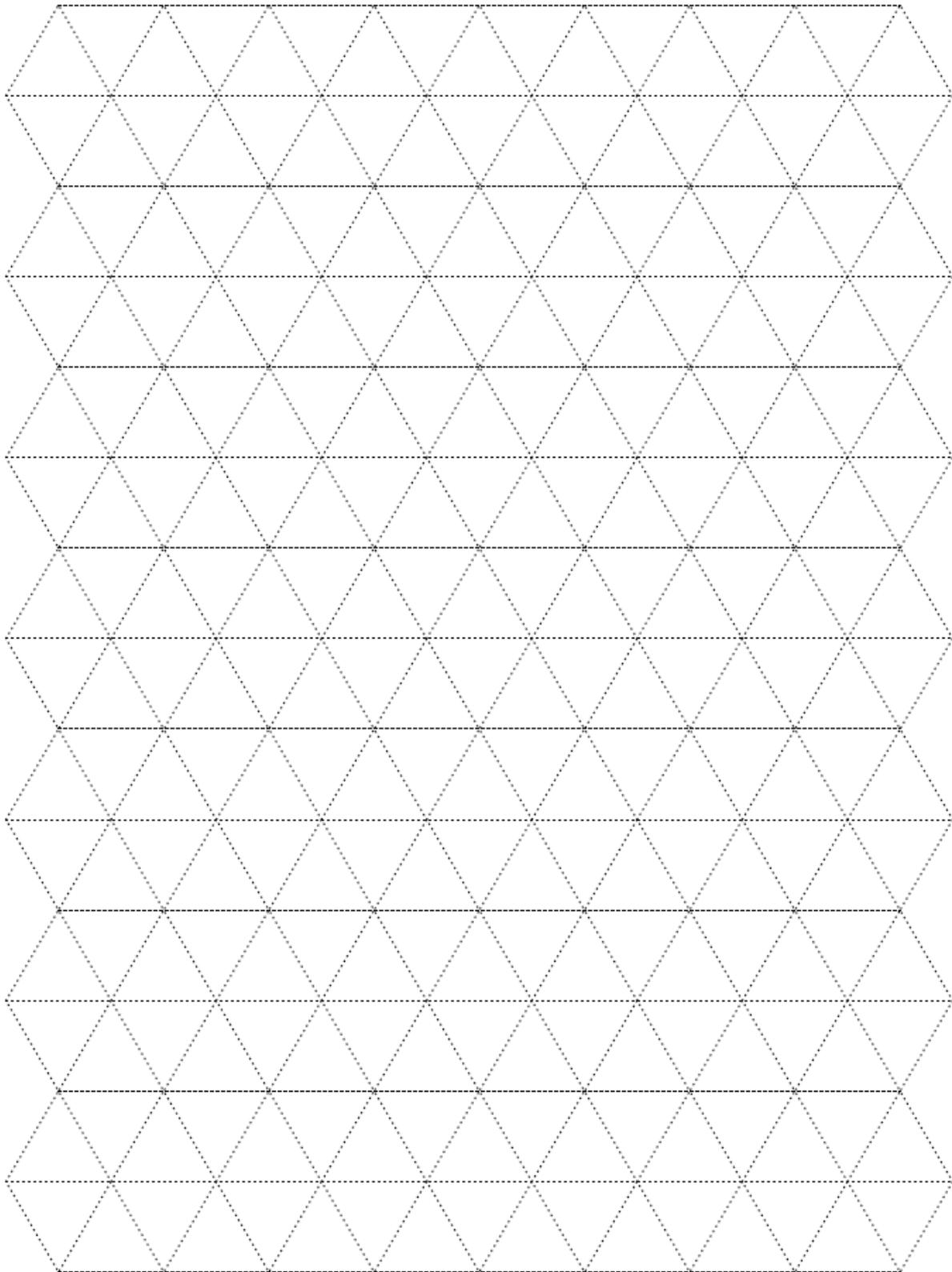
Culminating Task: Land Grant

STANDARDS FOR MATHEMATICAL CONTENT

MCC4.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
a. Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*
b. Understand a multiple of *a*/*b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

For this activity students will gain a lot of practice multiplying a whole number. In this particular task the whole number is 12. Students will also be asked to determine the remainder of the land after they completed the first two tasks in each problem. For example, the first problem states, "the remaining land would be left for gardening and a walking path". For this part of the problem students must also realize that they can subtract to determine that there are seven acres left. In addition to practicing these skills, students will organize their information in tables to better understand it, and make comparisons. They will be asked to write out number sentences or brief explanations to show their products (i.e. ¼ of 12 or ¼ x 12). Last, but not least, students will have to use the information they collected to make a sound argument for what plan is best for the community. Students need to have many opportunities to use math to not only justify their answers but also to justify their decisions in life. Adults do this constantly when we earn and spend. Students also need to know that math is often the deciding factor in many issues that we face today.

Before asking students to work on this task, be sure students are able to:
- Use repeated addition to add fractions with the same denominator.
- decompose fractions, for example ⁴⁄₄ = ½ + ½ or ¼ + ¾
- understand that the whole can be any number/size and the fractions always depend on taking a portion of this whole
- find equivalent fractions such as ⅙ = ²⁄₁₂.

ESSENTIAL QUESTIONS

- What does it mean to take a fraction portion of a whole number?
- How is multiplication of fractions similar to division of whole numbers?
- How do we determine the whole amount when given a fractional value of the whole?
- How do we determine a fractional value when given the whole number?

MATERIALS

- Colored Tiles
- Colored pencils or crayons
- Land Grant student recording sheet.

GROUPING

Individual/Partner Task

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students will decide how best to use a 12 acre plot of land that has been donated to the local community. Students will have to use what they have learned about addition, subtraction, and multiplication of fractions in order to successfully complete this task. In addition to this, students will also practice multiplying whole numbers and use amounts, such as acreage and building costs, to determine what plan is best for the community.

Comments
Color tiles should be available for students to use. Students may want to just pick out three colors, one color for each purpose outlined in the activity. For example, in the first scenario the local library wants to build another branch. The 12 acres would be used as following, the library itself would be ⅙ of the land or 2 acres; the playground would be ¼ of the land or 3 acres and the remaining ⁷⁄₁₂ or 7 acres would be used for the garden. If a student chose to have red tiles for the library, yellow tiles for the playground and green tiles for the garden than their model could look like the one below.
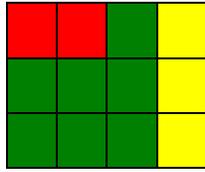
This task could be introduced by bringing in local newspaper articles or by looking at how land is being used in the local community. Blue prints and maps could also help students understand this task better. Land use is always a contentious issue and newspapers such as The Atlanta Journal Constitution constantly publish articles about how local communities are using land. Utilize www.ajc.com for more information.

Task Directions
   Students will follow directions below from the Land Grant task sheet. .
   - Obtain a set of colored tiles.
   - Draw each plan using colored pencils or crayons.
   - Work with a partner or small group to determine the products, equivalent fractions and/or sums for each step.
   - Complete the tables included in the student recording sheets.
   - Calculate some construction given a specific cost per acre.
   - Make an argument that utilizes math to help the city decide how to best use the land.

FORMATIVE ASSESSMENT QUESTIONS

   - How do you know how many acres the building, garden, pool, etc. will be?
   - What strategies did you use to determine your product? (i.e. equivalent fractions, division of whole numbers, repeated addition)
   - Do you think this is a good use of the land? Why or why not?
   - Is there another way that you can make this design work?

DIFFERENTIATION

   Extension
   - Once students have completed the task above, this lesson could be extended to use a larger acreage, such as 24, 30, or 36.
   - Students could use more practice solving problems were the numerator is a number other than 1. For example ⅚ of 30.
   - Students could solve more challenging problems that involve not just larger areas of land, but also more than three features on the land. For example a 24 acre piece of land that will house a sports field, recreation center, pool, tennis courts, basketball courts, parking lot, etc.

Intervention
- Students may need to review division and have experiences where they can see the relationship between the two.
- Students may use repeated addition to solve these problems.
- Students could begin with a smaller number of acres, such as 6 or 8 and work up to the task as it is written.
- Students could solve for just two parts of the problems, such as the buildings and parking lot.

Name: _____   Date: _____

Land Grant
Part 1
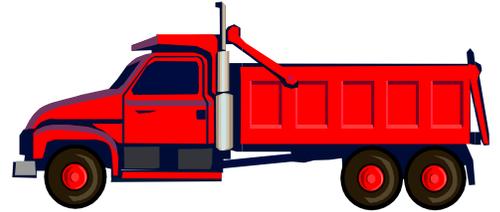
Someone recently donated 12 acres of land to the City Council. The donor wants the land to be used for the public good. Three different organizations have proposed plans for the land, below. The local government needs your help deciding how this land could be best used. Make a drawing of each plan and determine how many acres they will need for both their buildings and their landscapes.

1. The Local Library wants to build a new library and use the remaining land for a community garden and a playground. The Library would take up one-sixth of the land, the playground would take up one-fourth of the land and the remaining land would be left for the community garden and a walking path. Determine how many acres of land each for the building, the playground and the garden.

	Fraction of the Whole	**Acres**
Building		
Playground		
Garden		

2. The Parks and Recreation Department would also like to use the land, but for different purposes. They hope to use one third of the land for a Recreation Center that would include dance and fitness rooms, an art room, a gymnasium and locker rooms. In addition to this building, they also want to use one sixth of the land to build a swimming pool and tennis courts. Finally, they would use one half of the land for playing fields and a parking lot. Determine how many acres of land each for the Recreation Center, the pool, the tennis courts, and the playing fields.

	Fraction of the Whole	**Acres**
Recreation Center		
Swimming Pool & Tennis Courts		
Playing Fields and Parking Lot		

3. Last but not least, The National Forest Service wants to use the land to preserve the trees there. They would leave two-thirds of the land untouched but available for hiking trails. One fourth of the land would be used for a visitor center which would include a small museum and office. The remaining land would become a parking lot. Determine how many acres of land each for the hiking trails, the visitor center and the parking lot.
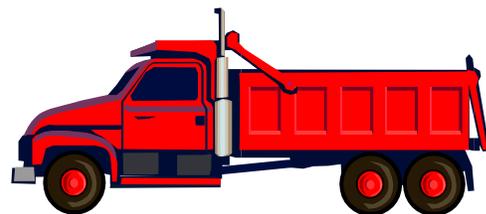
	Fraction of the Whole	**Acres**
Hiking Trails		
Visitor Center		
Parking Lot		

Name: _____   Date: _____

Land Grant
Part 2

The City Council has to provide running water and electricity to any building that is built. They use taxes to pay for this but it is expensive and will cost $1,200 per acre. Determine how much tax money the City Council will need for each of the plans.

Organization	Tax Money Required
Library	
Parks and Recreation Department	
National Forest Service	

Now that we know the details of each organizations proposal, including how much tax money each will cost the city, please decide which land proposal you think is best.

- You must include fractions in your answer
- You must include cost in your answer
- You must considered what is best for the community
- You may include any other information you feel is important

_____

_____

_____

_____

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_____

_____